Time-Evolution of Maritime Domain Awareness

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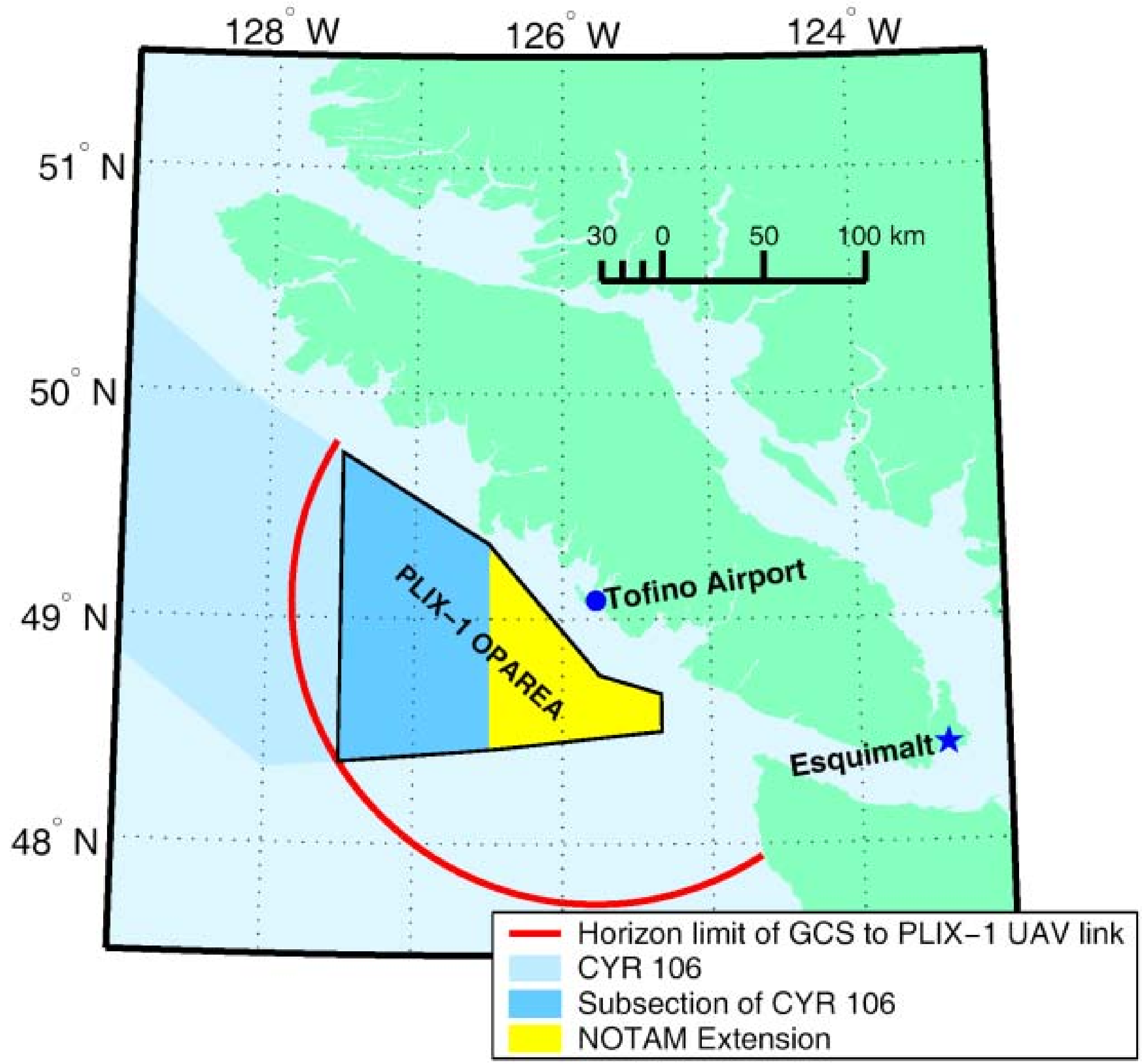
Maritime Domain Awareness

- Maritime Domain Awareness (MDA) depends upon Intelligence, Surveillance, and Reconnaissance (ISR)
- MDA is generated by completing ISR tasks that characterize contacts

ISR Experiment

- ISR in Maritime context
  - Littoral waters near Tofino, BC
- Live experiment, including UAV
- July 7 to 11, 2003



Dynamical IISRA Model (DIISRAM)

- Four contact characterization states:
  - Detection: x_1
  - length measurement: x_2
  - classification: x_3
  - identification: x_4
- Predict the time-evolution of $\mathbf{x} = (x_1, x_2, x_3, x_4)$
- nonlinear system:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$$

Postulate 1

- Capability Limit $\Rightarrow$ MDA evolves toward a steady-state

Postulate 2

- Object availability $\Rightarrow$ MDA evolution responds to the number of objects in each contact characterization state

Postulate 3

- Task Activation $\Rightarrow$ MDA evolution depends on the stability of precursory and/or competitive tasks

Postulate 4

- Capability overreach $\Rightarrow$ MDA evolution can temporarily exceed its steady-state limits

Postulates 1, 2, 3
Math Summary

- To *first order*, rates are proportional to
  - the difference from steady-state

$$\dot{x}_k \propto \left(a_k N - x_k\right)$$

  - the number of targets

$$\dot{x}_k \propto N$$

  - the stability of precursory or competitive processes

$$\dot{x}_k \propto \frac{x_j}{a_j N}, \quad j \neq k$$

Capability Overreach
Math Summary

- Steady-state can be temporarily exceeded

$$\dot{x}_k \propto \left(a_k N + g_k(\mathbf{a},\mathbf{x}) - x_k\right)$$

- Excess contacts lost during subsequent processing

$$g_k(\mathbf{a},\mathbf{x}) = \sum_{j\neq k} a_{m(j,k)} \frac{a_{i(j,k)}N - x_{i(j,k)}}{a_{i(j,k)}N - a_{n(j,k)}}$$

Best-Fit Solution

- Simple
- Text-book methods
  - Runge-Kutta solver
  - Downhill simplex search (Nelder & Mead)
  - Least squares
- Constraints: non-negative contact counts, not more than the number detected
- Penalty-function: unconstrained non-linear optimization

Best-Fit Solution
Math Summary

- Textbook Methods: Downhill Simplex Search & Least Squares

$$\chi^2 = \sum_{k=1}^{N_D} \left[\frac{\mathbf{F}(\mathbf{a}, \mathbf{x}(t_k)) \bullet \mathbf{u}_k - \mathrm{y}_k}{\sigma_k} \right]^2$$

- Numerical integration subject to constraints on characterization-state counts

$$0 \le x_k \le x_1$$

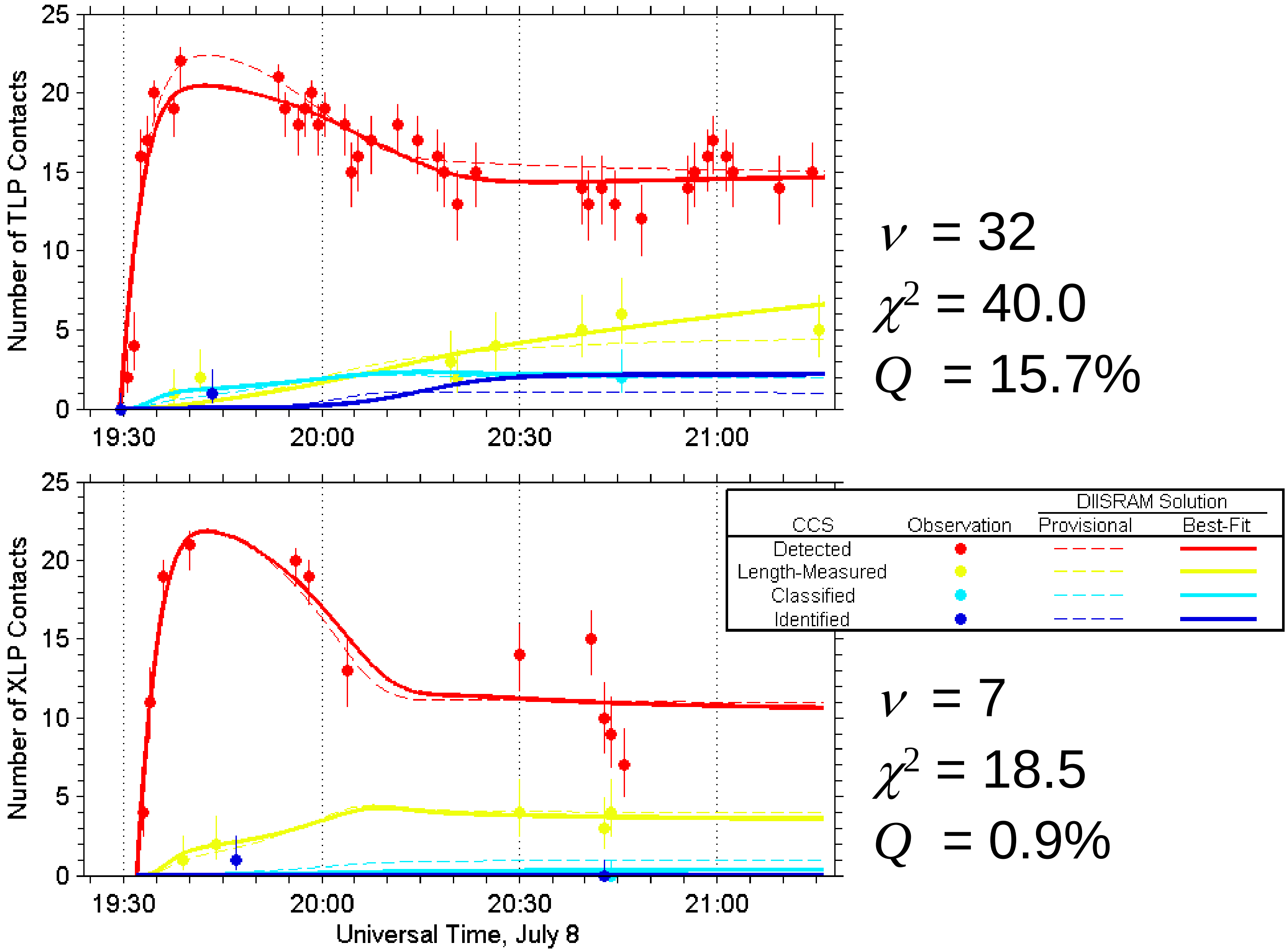
- Penalty-function checks constraints on 19 model parameters

$$P(\mathbf{a}) = \begin{cases} 0, & \mathbf{a} \in \Omega \\ P_{\max}, & \text{otherwise} \end{cases}$$

A Tale of Two Pictures

- Tofino Littoral Picture (TLP)
  - Tactical-level
  - Located at Tofino airport (UAV's base)
  - Closest node to airborne sensors
- Experimental Littoral Picture (XLP)
  - Operational-level
  - Located at Canadian Forces Base Esquimalt
  - Furthest node from airborne sensors

Case 1: Capability Overreach



$\nu = 32$

$\chi^2 = 40.0$

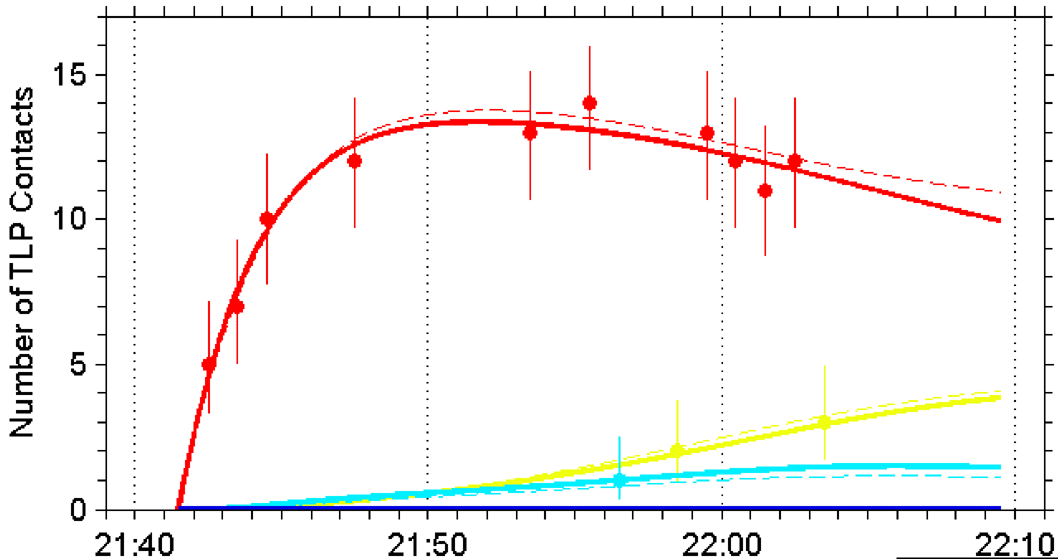
$Q = 15.7\%$

$\nu = 7$

$\chi^2 = 18.5$

$Q = 0.9\%$

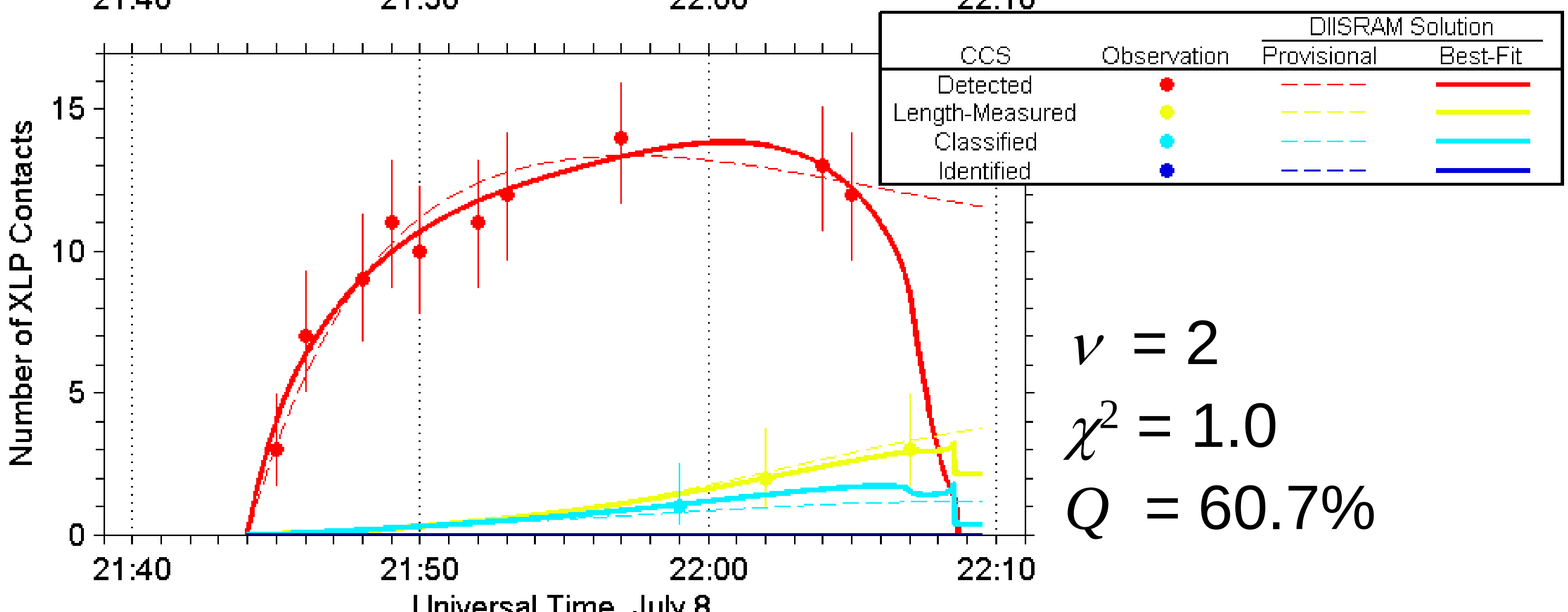
Case 2: Sensitive Solution



$\nu = 2$

$\chi^2 = 0.8$

$Q = 66.4\%$

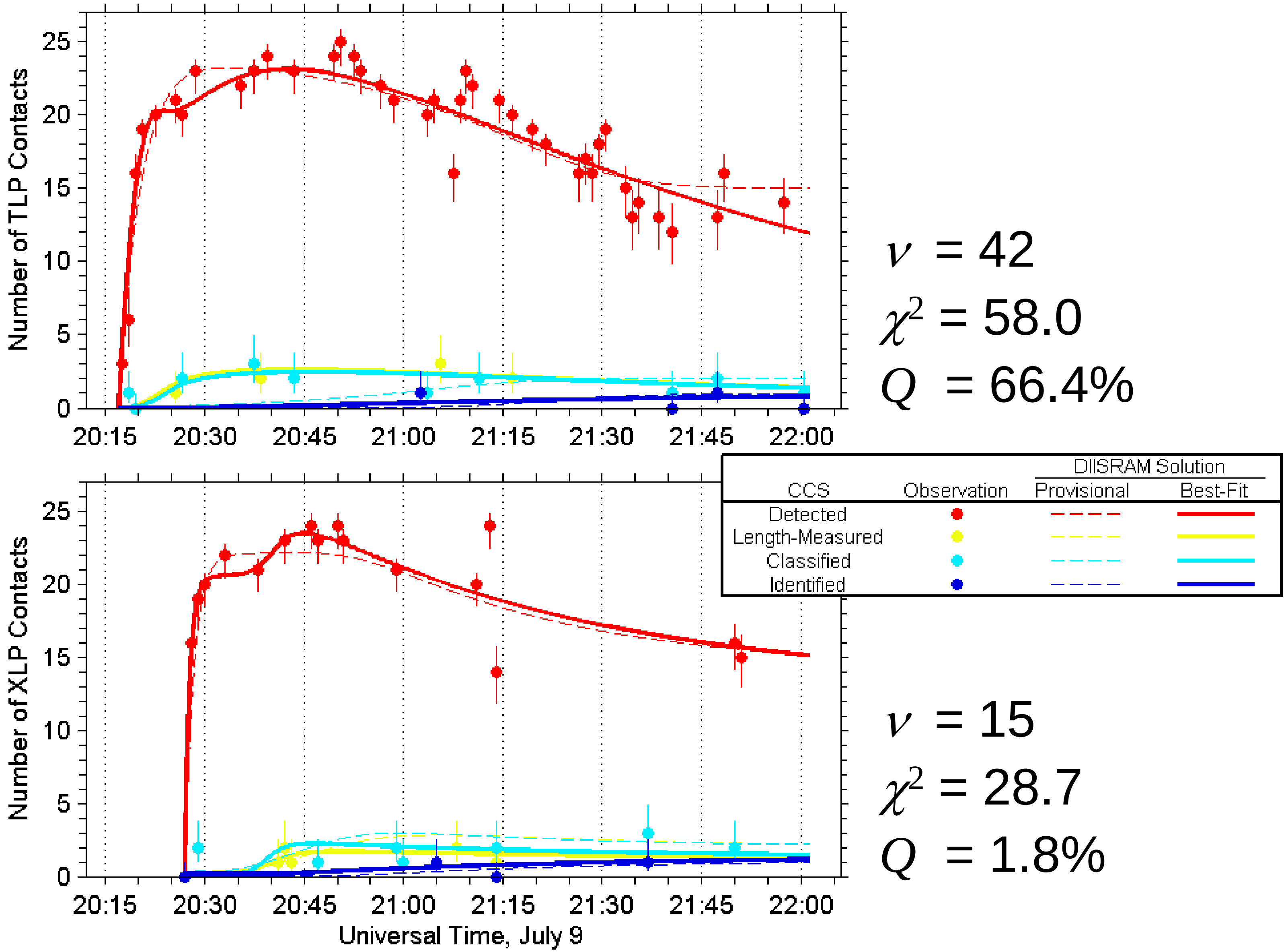


$\nu = 2$

$\chi^2 = 1.0$

$Q = 60.7\%$

Case 4: Capability Under-Reach



ν = 42

χ^2 = 58.0

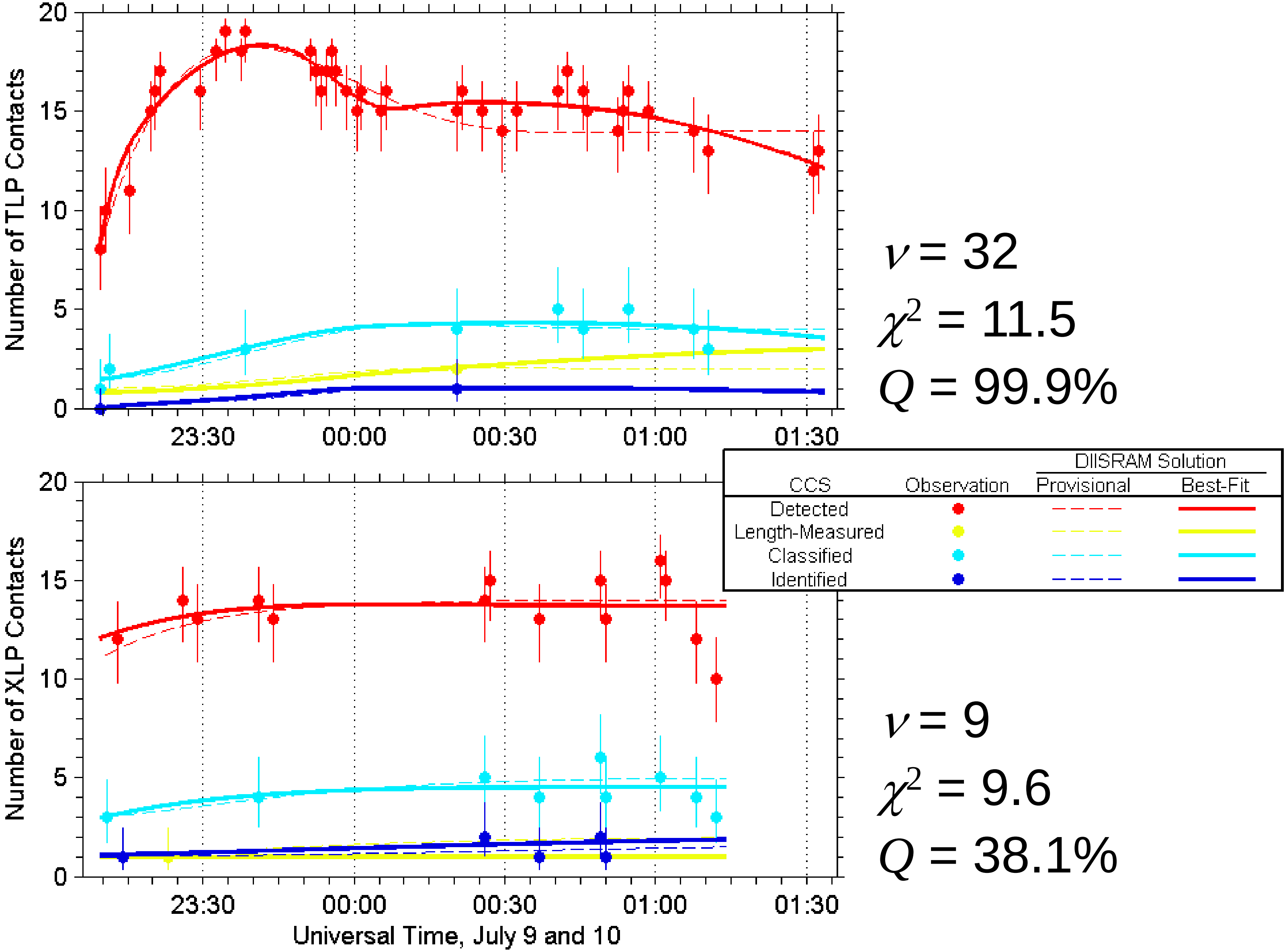
Q = 66.4%

ν = 15

χ^2 = 28.7

Q = 1.8%

Case 6: Double Overreach



$\nu = 32$

$\chi^2 = 11.5$

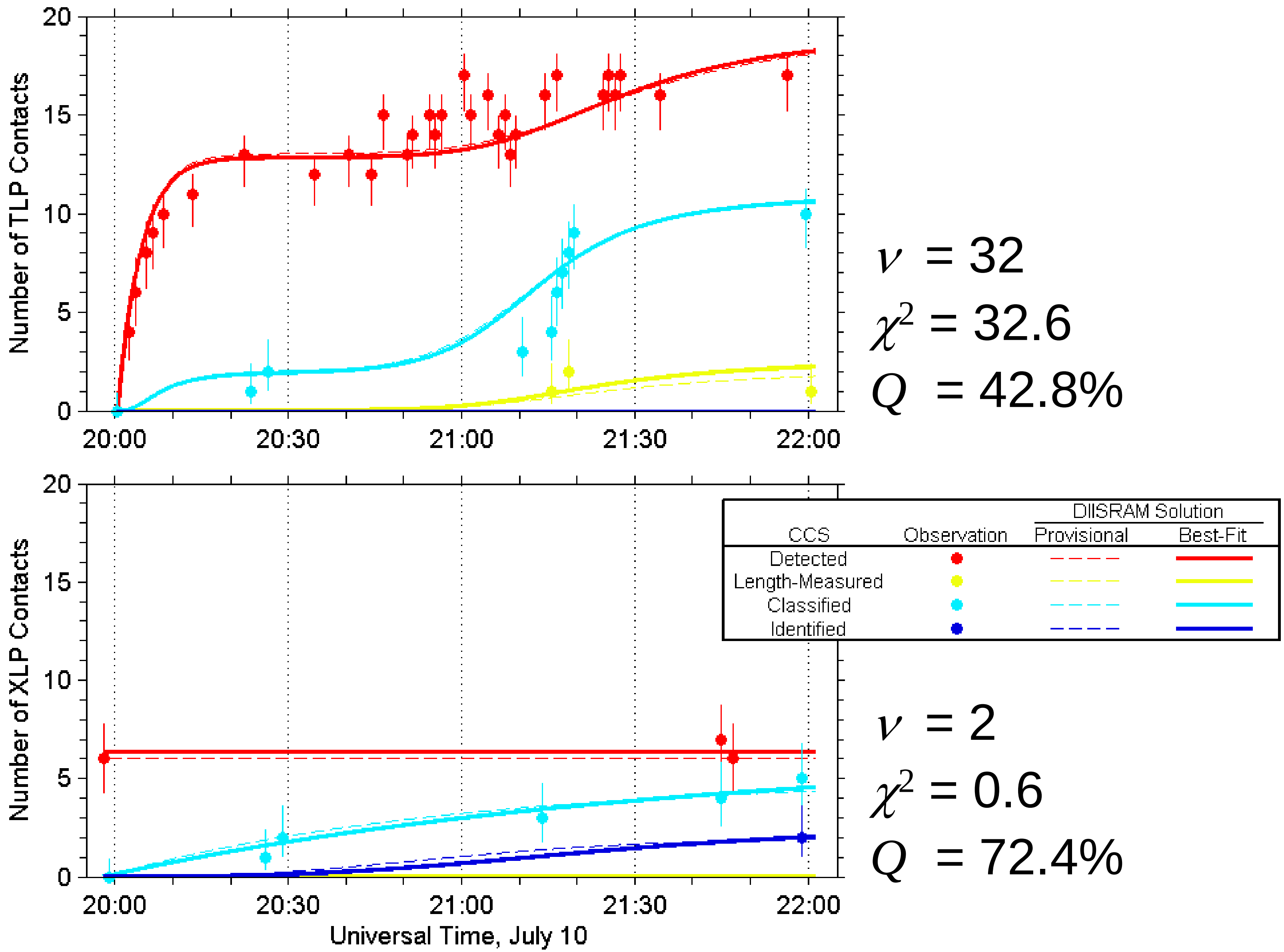
$Q = 99.9\%$

$\nu = 9$

$\chi^2 = 9.6$

$Q = 38.1\%$

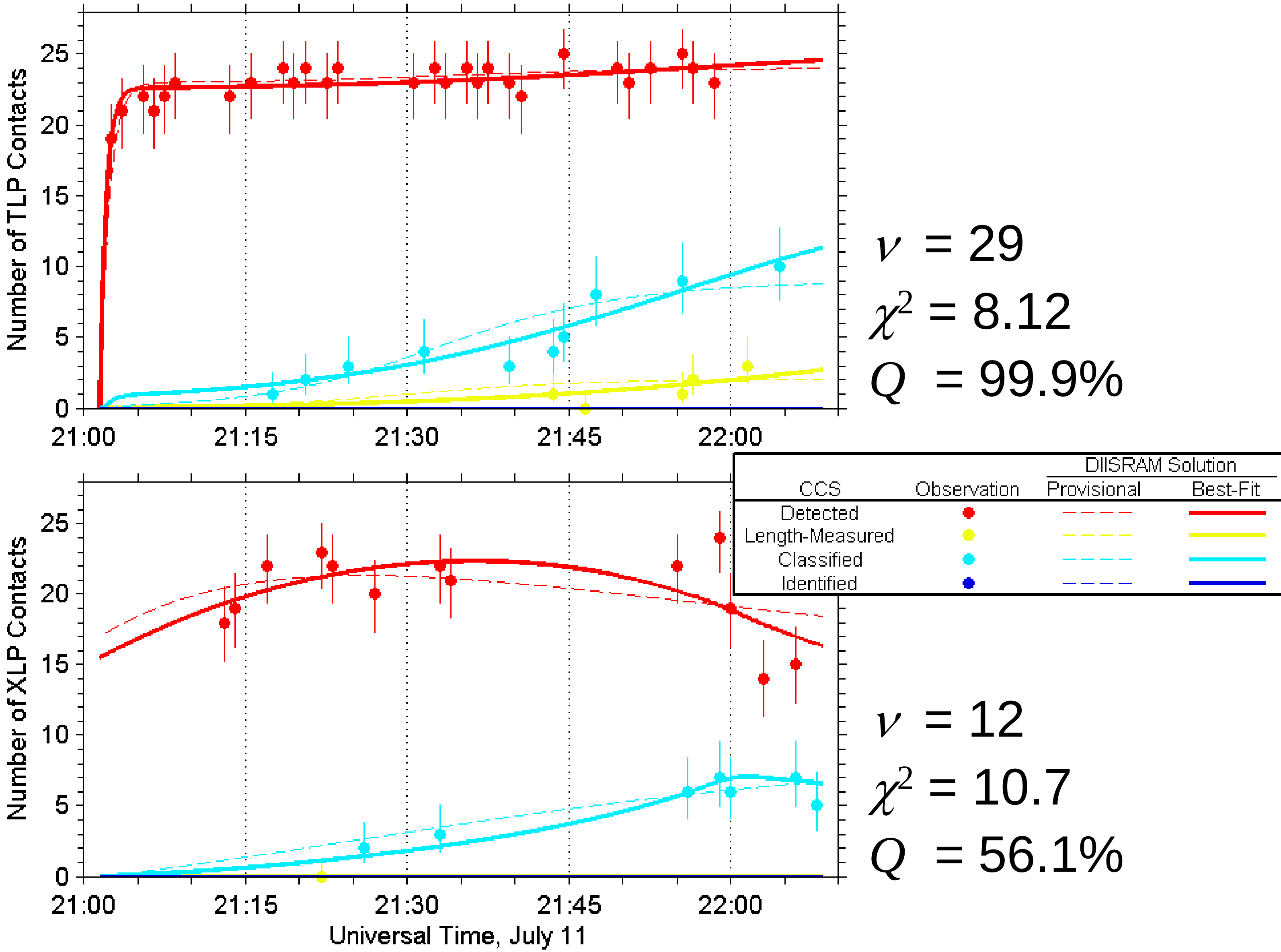
Case 9: Large Under-Reach



ν = 32
χ^2 = 32.6
Q = 42.8%

ν = 2
χ^2 = 0.6
Q = 72.4%

Case 12: Inversion of Reach



$\nu = 29$

$\chi^2 = 8.12$

$Q = 99.9\%$

$\nu = 12$

$\chi^2 = 10.7$

$Q = 56.1\%$

Results Summary
Tofino Littoral Picture (TLP)

Period	Critical Numbers			DIISRAM Solution: Provisional		DIISRAM Solution: Best-Fit	
	Data Points N_D	DIISRAM Parameters N_P	Degrees of Freedom ν	Reduced Chi-Squared χ^2/ν	Confidence Level $Q(\chi^2,\nu)$ %	Reduced Chi-Squared χ^2/ν	Confidence Level $Q(\chi^2,\nu)$ %
1	52	20	32	1.57	2.2	1.25	15.7
2	14	13	2	0.64	52.4	0.40	66.4
3	19	15	5	6.64	0.3	0.96	44.0
4	62	20	42	2.35	0.0002	1.38	5.2
5	23	20	6	3.01	0.6	1.70	11.7
6	52	20	32	0.60	96.0	0.36	99.9
7	16	15	1	3.25	1.3	2.19	6.7
8	39	16	25	0.84	68.8	0.60	94.3
9	46	16	32	1.05	39.1	1.02	42.8
10	48	10	39	1.54	1.7	1.39	5.4
11	42	15	28	1.88	0.3	1.09	34.2
12	43	15	29	0.52	98.5	0.28	99.9

Results Summary
Experimental Littoral Picture (XLP)

Period	Critical Numbers			DIISRAM Solution			
				Provisional		Best-Fit	
	Data Points N_{D}	DIISRAM Parameters N_{P}	Degrees of Freedom ν	Reduced Chi-Squared χ^2/ν	Confidence Level $Q(\chi^2,\nu)$ %	Reduced Chi-Squared χ^2/ν	Confidence Level $Q(\chi^2,\nu)$ %
1	21	14	7	2.81	0.6	2.64	0.9
2	14	12	2	0.93	39.6	0.50	60.7
3	16	12	4	0.86	48.7	0.25	90.8
4	35	20	15	2.92	0.01	1.91	1.8
5	7	6	1	2.85	9.1	1.54	21.4
6	29	20	9	1.21	28.3	1.07	38.1
7	9	7	2	4.70	0.9	3.60	2.7
8	5	3	2	1.52	21.9	1.14	31.9
9	10	81	2	0.51	60.1	0.32	72.4
10	17	9	8	2.97	0.03	2.89	0.3
11	20	9	11	0.91	52.5	0.75	68.6
12	21	9	12	1.09	36.2	0.89	56.1

Conclusions

- Goodness-of-fit statistics indicated that the model's solutions were acceptable in 20 out of 24 cases
  - Acceptable for 100% of TLP cases and 67% of XLP cases
  - The solution emulated the multi-state count collapse (Cases 2 and 8)
  - Capability overreach was observed, including one double overreach (TLP Case 6)
  - Capability under-reach (opposite of overreach) was discovered (Cases 4 and 9)
  - Inverted a capability under-reach in the TLP into an overreach in the XLP (Case 12)

Practical Recommendation

- Apply the model to other real configurations of ISR assets
  - Assess goodness of fit
  - Empirical parameters would enable quantitative predictions of the time-evolution of live ISR operations
    - In other words, the model would aid MDA/ISR force planning & development

Questions?